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Degenerate Dynamical System Normal Form and the Oppenheimer-Snyder model

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Abstract

Degenerate dynamical systems (DDS) arise when the geometric structure governing evolution loses rank along subsets of phase space. As emphasized by V. Arnold (1), the apparent arbitrariness largely disappears once degeneracy is organized by intrinsic geometric mechanisms. Extending the Saavedra-Troncoso-Zanelli analysis of regular hypersurfaces of constant corank (4), we develop a unified intrinsic framework for smoothly stratified degeneracies, including Morse-type walls meeting transversely. Under assumptions of smoothness, clean intersection, and fixed corank, we obtain local normal forms for presymplectic structures, in which admissible degeneracies split into universal elementary blocks determined by transverse degeneracy coordinates and tangential presymplectic geometry. Closed correction forms encode residual geometric data whose relative cohomology classes yield intrinsic local invariants obstructing the equivalence of DDS with the same degeneracy pattern (2).

The theory is illustrated on the Oppenheimer-Snyder model provides one of the simplest exact descriptions of gravitational collapse leading to black-hole formation (3).

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References

[1] V. I. Arnold, A. B. Givental, and A. Weinstein, Symplectic geometry, in *Dynamical Systems IV*, Encyclopaedia of Math Sc, Vol. 4, Springer, 2001.

[2] V. Ayala and C. Kawan, Topological Conjugacy of Real Projective Flows. Journal of the London Mathematical Society, (2), 90, (2014), 49-66.

[3] J. R. Oppenheimer and H. Snyder, On continued gravitational contraction, *Physical Review* **56** (1939), 455–459. doi: 10.1103/PhysRev.56.455.

[4] R. Saavedra, T. Troncoso, and J. Zanelli, Degenerate dynamical systems, *Phys. Rev. D* **102** (2020), 085014.

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Divisor de ramificación reducible y anillos de Cox de superficies K3

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Resumen

Una de las técnicas para calcular anillos de Cox, es usando ciertos cubrimientos finitos $\pi : X \to Y$ para describir el anillo de Cox de X en términos del anillo de Cox de Y.
Para superficies K3 génericas con una involución no simpléctica $\sigma : X \to X$, el mapeo cuociente asociado $\pi : X \to Y = X/\langle\sigma\rangle$ es un cubrimiento doble. Si el cubrimiento es no-ramificado entonces $Y = X/\langle\sigma\rangle$ es una superficie de Enriques, en otro caso Y es una superficie racional suave y el divisor de ramificación $B \in \mathrm{WDiv}(Y)$ satisface $B \sim -2K_Y$.
El caso en que B es irreducible ($n = 1$) o tiene dos componentes conexas ($n = 2$) fue estudiado en profundidad por Artebani, Hausen y Laface [1], quienes dan una presentación explícita de el anillo de Cox de X en términos del anillo de Cox de Y. El caso $n \geq 3$ permanece abierto.

En esta charla presentaré resultados sobre el caso $n = 3$. Un análisis de la tabla de la Sección 16 del Atlas de Roulleau [2] muestra que este caso ocurre para exactamente cuatro familias de superficies K3, con reticulado de Néron–Severi isométrico a alguno de los siguientes, para número de Picard $\rho \in \{9, 10, 11\}$:

$$U \oplus D_4 \oplus A_1^3 \ (\rho = 9), \quad U \oplus D_4 \oplus A_1^4 \ (\rho = 10), \quad U(2) \oplus D_4^2 \ (\rho = 10), \quad U \oplus D_4 \oplus A_1^5 \ (\rho = 11).$$

Referencias

[1] M. Artebani, J. Hausen, A. Laface, *On Cox rings of K3 surfaces*, Compositio Math. **146** (2010), no. 4, 964–998.

[2] X. Roulleau, *An atlas of K3 surfaces with finite automorphism group*, Épijournal Géom. Algébrique **6** (2022), article 19.

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Weak conditions for existence of control sets

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Abstract

In this talk, we establish weak conditions for the existence of a control set with nonempty interior for a linear control system on a solvable Lie group. We show that the Lie algebra rank condition, together with the compactness of the nilpotent part of the generalized kernel of the drift, is sufficient to guarantee the existence of such a control set. Moreover, this control set is unique and contains the entire generalized kernel in its closure.

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On Induced Linear Control Systems over the Homogeneous Spaces of $SE(3)$

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Abstract

The special Euclidean group $SE(3) \cong \mathbb{R}^3 \rtimes SO(3)$ is the fundamental state space for rigid body motions, playing a central role in the geometric analysis of robotic kinematics and underactuated spatial systems. The controllability of Linear control systems (abbrev. LCSs) is well-established on macroscopic quotient spaces, yielding full-dimensional control sets on the pure positional space $\mathbb{R}^3$ and rotational space $SO(3)$. However, the geometric behavior induced by lower-dimensional subgroups remains largely unexplored. Building on Jouan's foundational framework for the equivalence of control systems with LCSs on Lie groups and homogeneous spaces [2], this presentation investigates the induced linear dynamics on the lower-dimensional quotients of $SE(3)$. The discussion will focus on the geometric conditions required for the existence of control sets with nonempty interiors, ultimately revealing the fundamental topological obstructions inherent to these spatial state spaces. For further details regarding this theoretical basis and the application of this methodology to other Lie groups, see [1] and [3], along with the references therein.

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References

[1] Fritz Colonius and W. Kliemann, *The Dynamics of Control*. Birkhauser, Boston; 2000.

[2] Philippe Jouan, *Equivalence of control systems with linear systems on Lie groups and homogeneous spaces*. ESAIM: Control, Optimisation and Calculus of Variations; 16(4), 956-973 (2010).

[3] Adriano Da Silva, Okan Duman, Eyup Kizil, *One-input linear control systems on the homogeneous spaces of the Heisenberg group-the singular case*. Journal of Differential Equations; 407, 133-152 (2024).

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Superficies $K3$ Mori dream con singularidades y número de Picard 2

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Resumen

En esta charla introduciré las superficies $K3$ con singularidades y presentaré algunos criterios para determinar cuándo son Mori dream. Mostraré que, si el retículo de Picard de una superficie $K3$ con singularidades es Mori dream, entonces la superficie también es Mori dream. Además, para superficies $K3$ con singularidades y número de Picard dos, veremos que ser Mori dream es equivalente a contener dos curvas negativas que se intersectan entre sí. Finalmente, presentaré una clasificación de algunas superficies $K3$ con singularidades y número de Picard dos a partir de su matriz de intersección.

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Referencias

[1] A. Laface, A. Massarenti and W. D. Montoya, *Mori dream singular K3 surfaces*. Mathematische Zeitschrift **310**, 61 (2025).

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The Stability of Linear Control Systems on Low-Dimensional Lie Groups

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Abstract

Internal stability, asymptotic internal stability, and BIBO stability have been extensively studied in the context of linear systems on Euclidean spaces, where these properties are completely characterized by the spectrum of the *drift* of the system and the output map; see [1]. These notions can be extended to Lie groups. In [2], a characterization of these stability properties is proposed for linear control systems on connected matrix Lie groups.
Consider a linear control system Σ_G on a connected Lie group G, given by

$$\Sigma_G : \dot{g}(t) = \mathcal{X}(g(t)) + \sum_{i=1}^{m} u_i(t) Y_i(g(t)), \qquad g(t) \in G, t \in \mathbb{R}, u \in \mathcal{U}$$

where $\mathcal{X}$ is a linear vector field and the control fields $Y_i \in \mathfrak{g}$ are left-invariant vector fields.
The system is said to be:

1. *Internally stable (IS)* if
$$\text{the orbit } \{\varphi_t(g)\}_{t\in\mathbb{R}^+} \text{ is bounded for every } g \in G.$$
2. *Asymptotically internally stable (AIS)* if
$$\varphi_t(g) \to e, \forall g \in G.$$
3. *Internally unstable* if it is not internally stable.
4. *Bounded input - bounded output stable (BIBO - stable)* if, given a Lie group homomorphism $h : G \to H$, for every bounded input $u \in \mathcal{U}$, that is, $|u(t)| \leq M, \forall t > 0$, the set
$$\{h(\varphi(e, u, t))\}_{t\in\mathbb{R}^+}$$
is bounded.

It has been conjectured that the qualitative behavior of solutions of linear control systems, and in particular their stability properties, is determined by the spectral decomposition of the derivation associated with the drift $\mathcal{X}$, together with the algebraic structure of the underlying Lie group.
In this presentation we investigate these notions of stability through three low-dimensional examples:

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1. The two-dimensional Abelian group, whose stability properties are analyzed in both the connected and simply connected cases. In this setting, asymptotic internal stability is never achieved; however, internal stability holds under suitable conditions, while BIBO stability depends on the behavior of one component of the orbit and on the output map.
2. The two-dimensional solvable group, which can never be asymptotically internally stable. In this case, internal stability depends on a system parameter, whereas BIBO stability depends on the structure obtained by quotienting the Lie group by a discrete subgroup.
3. The three-dimensional Heisenberg group, whose internal stability is closely related to the spectral properties of the derivation associated with the drift. Regarding BIBO stability, it depends on the output map and on the associated discrete normal subgroups; moreover, in certain cases, the spectrum of the derivation also plays a role.

The examples considered highlight the influence of the spectrum of the derivation associated with the drift $\mathcal{X}$, the output map h, and the algebraic structure of the underlying Lie group on the stability properties of the system [3].

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References

[1] Lewis, A.D *A Mathematical Approach to Classical Control*. Preprint 2003. Available online: https://mast.queensu.ca/~bahman/mthe332/course-notes.pdf

[2] Ayala, V.; Torreblanca, M.; Valdivia, W. *Conjectures on the Stability of Linear Control Systems on Matrix Lie Groups*. Symmetry 2025, 17, 593. https://doi.org/10.3390/sym17040593

[3] Ayala, V.; Valdivia Hanco, W.E.; Pariapaza Mamani, J.E.; Todco, M.L.T. *The Stability of Linear Control Systems on Low-Dimensional Lie Groups*. Symmetry 2025, 17, 1766. https://doi.org/10.3390/sym17101766

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Estratos de Maxwell en el problema sub-Riemanniano en grupos de Lie de dimensión tres regulares, solubles y no nilpotentes

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Abstract

En este trabajo se estudia el problema sub-Riemanniano asociado a estructuras de contacto sobre grupos de Lie de dimensión 3 conexos, simplemente conexos, solubles, no nilpotentes y regulares. Para estos grupos, la componente vertical del sistema Hamiltoniano tiene la forma de un péndulo perturbado. Mediante un análisis cualitativo del espacio de fases, se demuestra que dicha componente vertical tiene simetrías no triviales. En particular, se caracteriza completamente el conjunto de Maxwell correspondiente a estas simetrías y se prueba que el primer tiempo de Maxwell coincide con el período del péndulo para casi toda geodésica. Este resultado proporciona una cota superior explícita para el tiempo de corte en términos del período del péndulo.

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References

[1] V. Ayala, A. Da Silva and D. A. García Hernandez, *Almost-Riemannian Structures on non-nilpotent, solvable 3D Lie groups*, Journal of Geometry and Physics 192, p. 1-15, 2023.

[2] A. Agrachev, D. Barilari and U. Boscain, *A comprehensive introduction to sub-Riemannian geometry*, Cambridge Stud. Adv. Math., vol. 181, Cambridge Univ. Press, Cambridge 2019.

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[3] I. Moiseev and Yu. L. Sachkov *Maxwell strata in sub-Riemannian problem on the group of motions of a plane* ESAIM Control Optim. Calc. Var. 16:2 (2010) 380-399.

[4] Ya. A. Butt, Yu. L. Sachkov and A. I. Bhatti *Maxwell strata and conjugate points in the sub-Riemannian problem on the Lie group* SH(2) J. Dyn. Control Syst. 22:4 (2016), 747â€“770.

[5] Ya. A. Butt, Yu. L. Sachkov and A. I. Bhatti *Cut locus and optimal synthesis in sub-Riemannian problem on the Lie group* SH(2) J. Dyn. Control Syst. 23:1 (2017), 155â€“195.

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Conjectures on the stability of Linear Control Systems on Matrix Lie Groups

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Abstract

The stability of control systems is fundamental to their reliable operation, ensuring that small variations in inputs, initial conditions, or parameters do not produce significant changes in system behavior. While several stability notions, including internal, asymptotic, and BIBO (Bounded Input, Bounded Output) stability, are well understood for linear control systems in Euclidean spaces, their counterparts on Lie groups remain largely unexplored.

This work addresses the stability problem for a class of linear control systems defined on matrix Lie groups, which generalize classical linear systems and reduce to them when the underlying group is Abelian and noncompact. We show how key algebraic, topological, and dynamical concepts used in the Euclidean setting can be extended to connected matrix Lie groups.

Building upon classical stability theory and the well-established notions of controllability and observability for linear systems on Lie groups, we formulate a set of conjectures concerning stability in this broader framework.

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References

[1] Agrachev, A. A., and Sachkov, Y., *Control theory from the geometric viewpoint* (Vol. 87). Springer Science & Business Media; 2013.

[2] Ayala, V., and Da Silva, A., *On the characterization of the controllability property for linear control systems on nonnilpotent, solvable three-dimensional Lie groups.* Journal of Differential Equations, 266(12), 8233-8257; 2019.

[3] Jouan, P., *Equivalence of control systems with linear systemson Lie groups and homogeneous spaces.* ESAIM: Control, Optimisation and Calculus of Variations, 16(4), 956-973; 2010.

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Weighted projective degenerations of $\mathbb{P}^n$

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Resumen

It is known by work of Manetti, Hacking and Prokhorov [3, 2] that any mildly singular degeneration of $\mathbb{P}^2$ is (a partial smoothing of) a weighted projective plane. We try to study the same question for higher dimensions, and find new examples of weighted projective spaces that are degenerations of $\mathbb{P}^n$. We use this to address questions on moduli of hypersurfaces and complete intersections. This is joint work with Kristin DeVleming and Jennifer Li.

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Referencias

[1] K. DeVleming, J. Li and S. Torres, *Weighted projective degenerations of* $\mathbb{P}^n$. Preprint

[2] P. Hacking and Y. G. Prokhorov, *Smoothable del Pezzo surfaces with quotient singularities.* Compos. Math. **146** (2010), no. 1, 169–192

[3] M. Manetti, *Normal degenerations of the complex projective plane.* J. Reine Angew. Math. **419** (1991), 89–118

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